Transformation Notes--Absolute Value

1. If absolute value brackets are around the entire function f(x), all negative y-values become positive y-values.

Example: y = | f(x) |

Quadrants 3 and 4 will move onto Quadrants 1 and 2.
All x-axis points will stay where they are

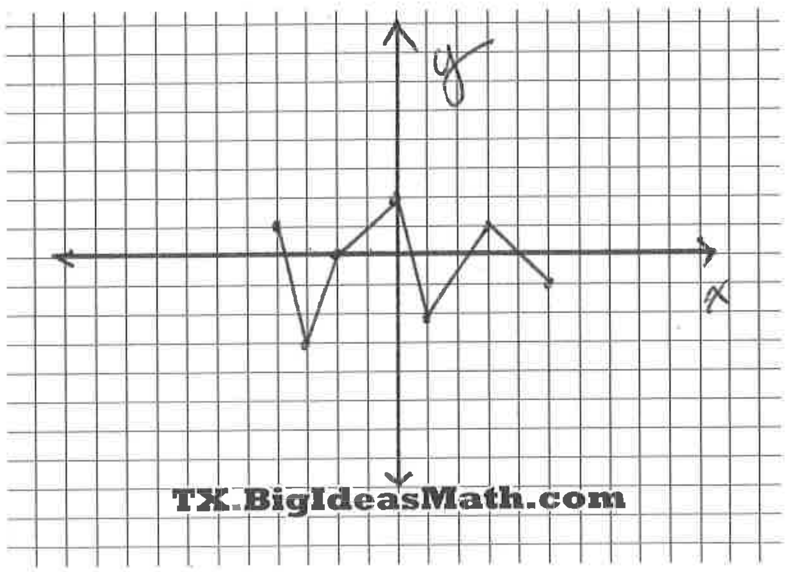
2. If absolute value brackets are around the x in the function f(x), the y-values on the right of the y-axis will repeat on the left side of the y-axis, and the old Quadrant 3 and 4 points will go away.

Example: y = f(| x |)

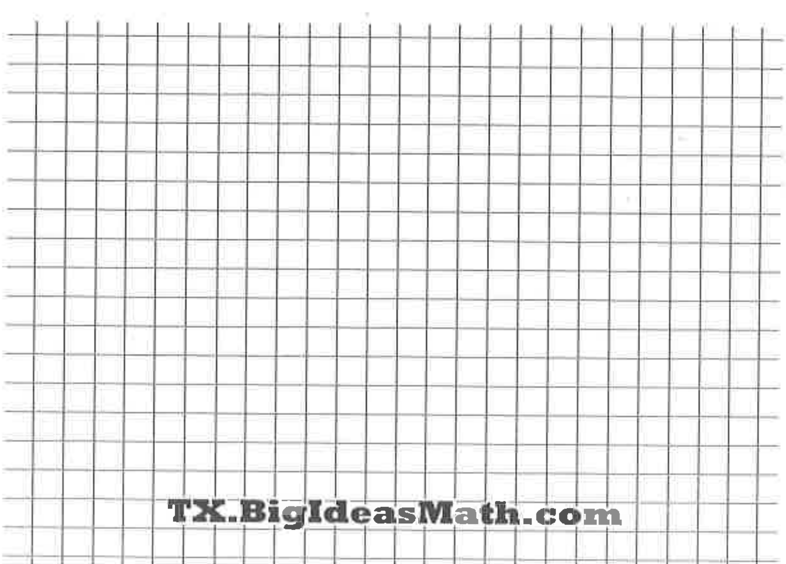
Quadrants 1 and 4 will repeat over the y-axis in Quadrants 2 and 3, causing old points from Quadrants 2 and 3 to disappear.

Absolute Value:

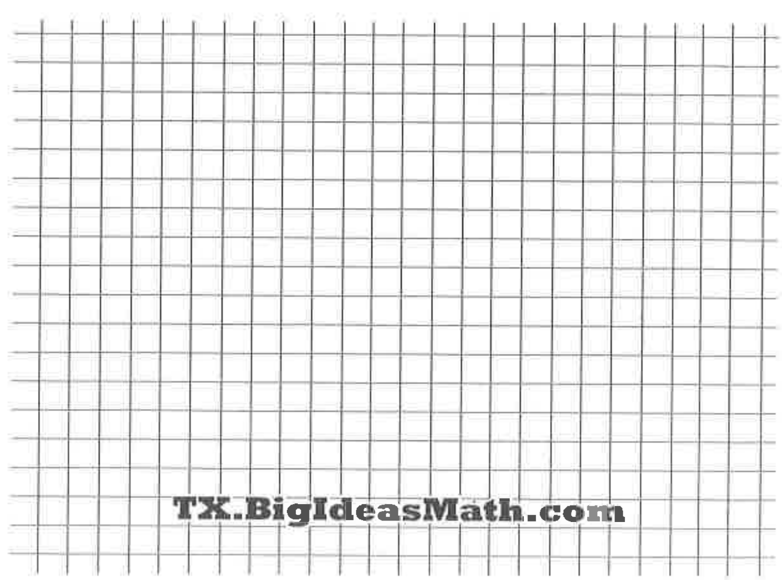
Given f(x)



1. graph y = | f(x) |

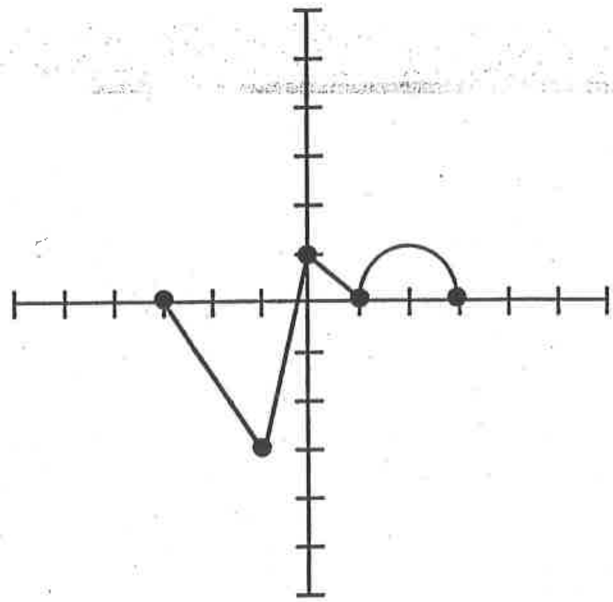


2. graph y = f(| x |)



Activity: Move the Monster

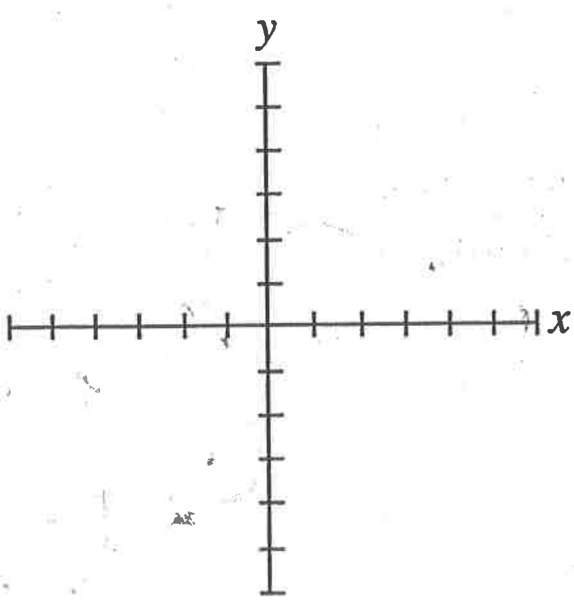
Given : $h(x)$

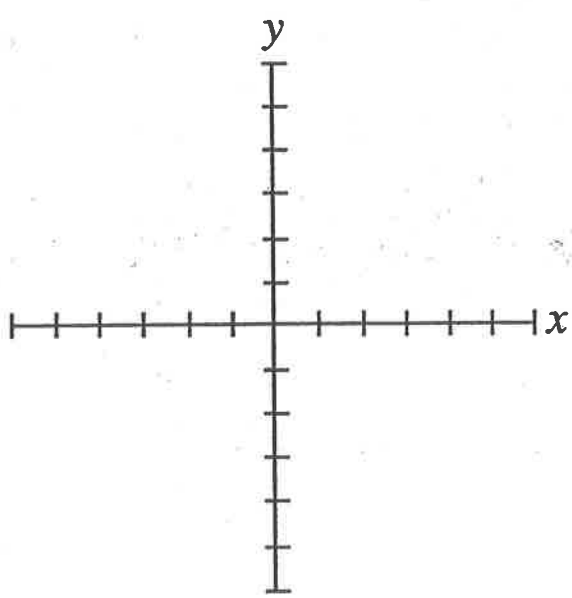


Sketch the graph of:

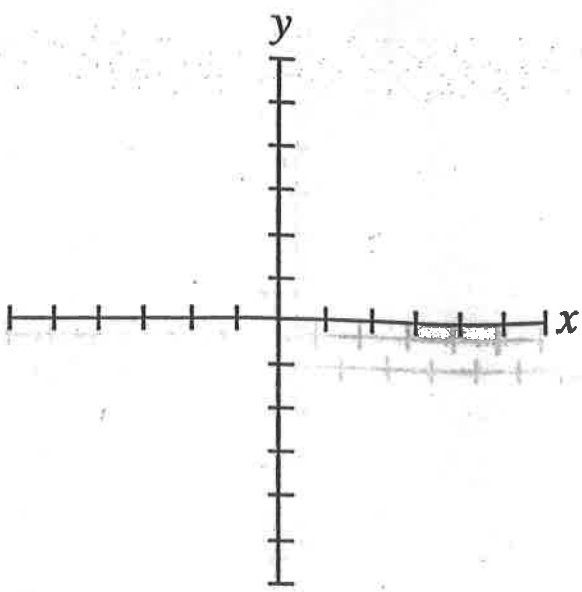
1. $-h(x)$

2. $h(-x)$

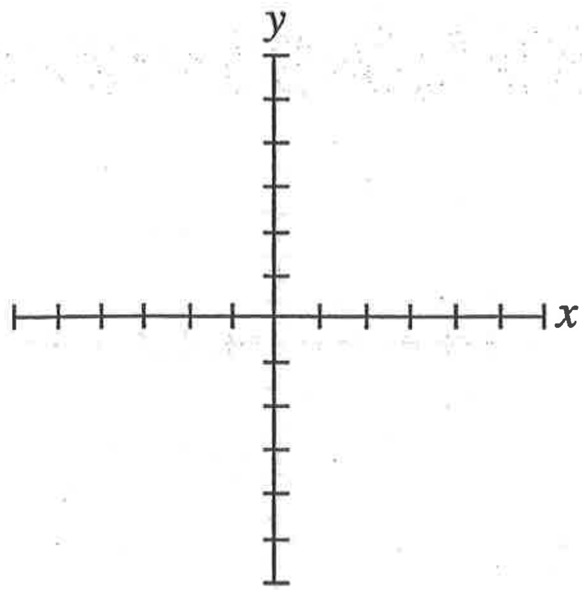




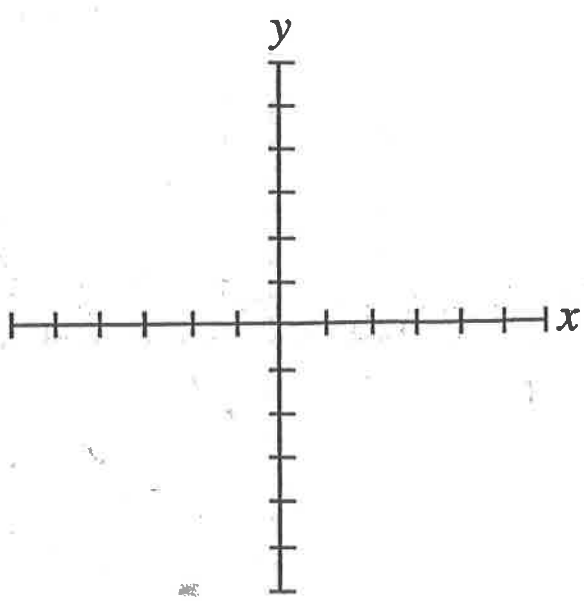
3. $h(x)+2$



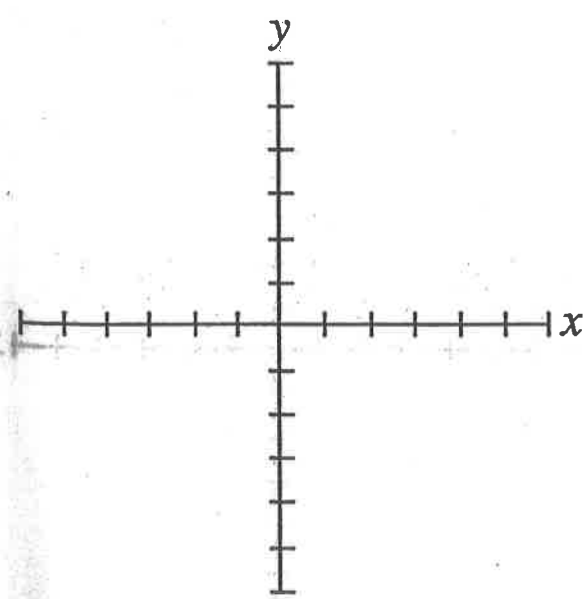
4. $h(x)-2$



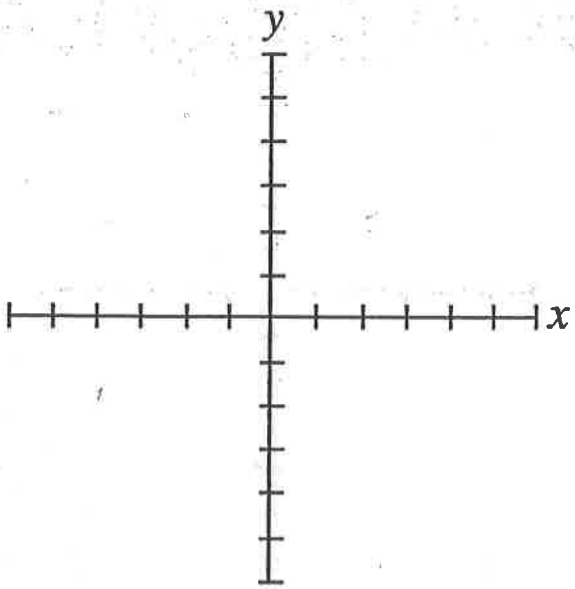
5. $h(x+2)$



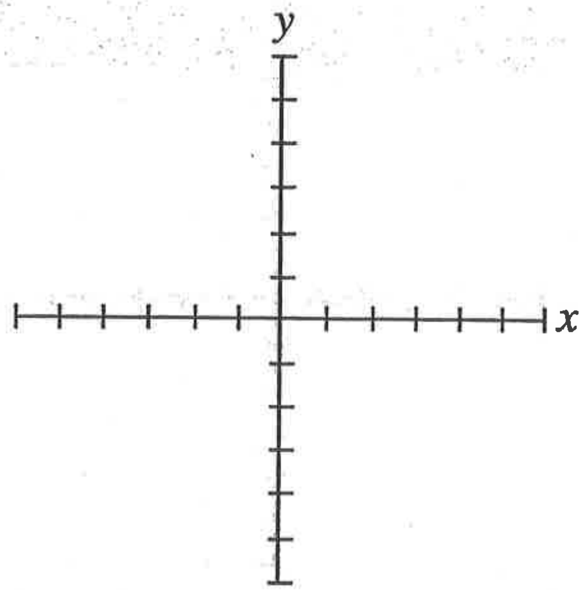
6. $h(x-2)$



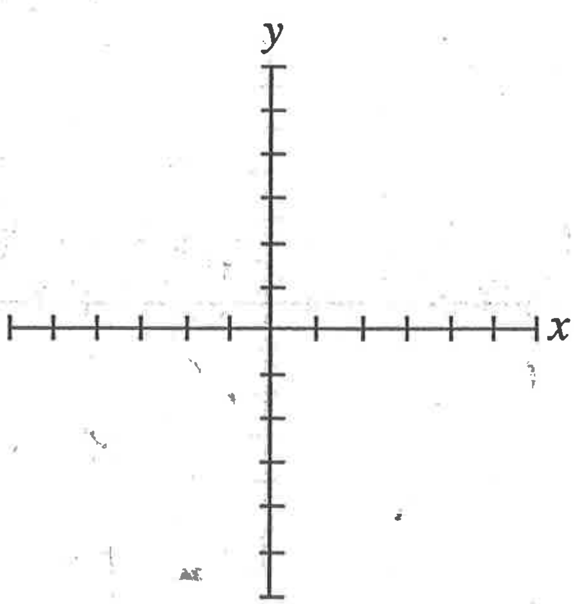
7. $2h(x)$



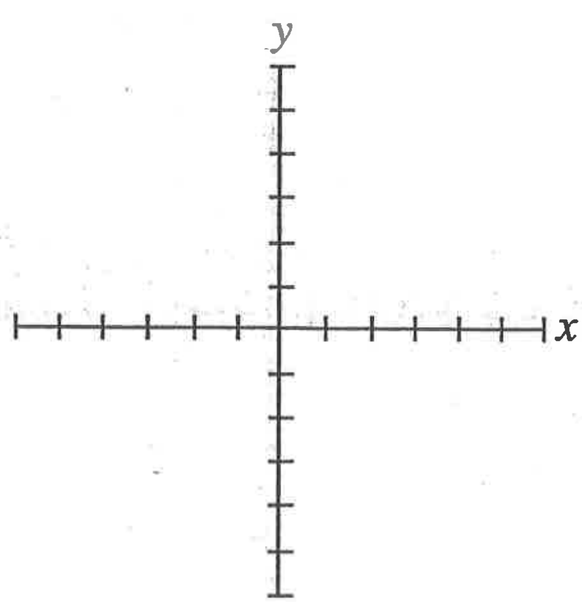
8. $\frac{1}{2}h(x)$



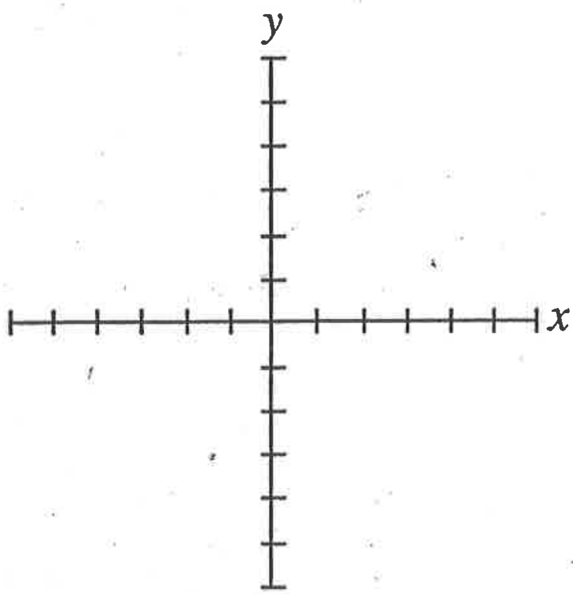
9. $h(2x)$



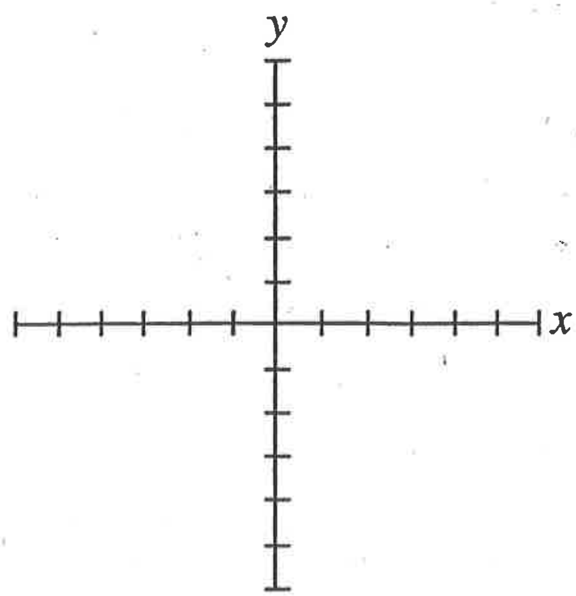
10. $h\left(\frac{1}{2}x\right)$

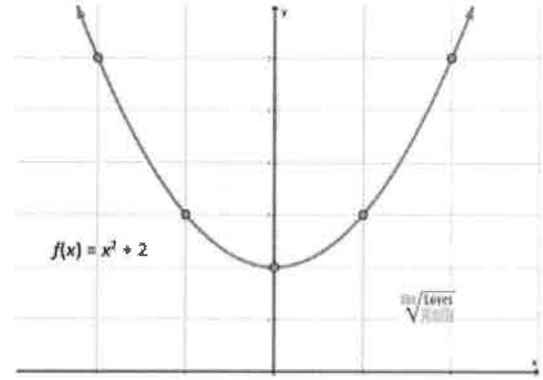
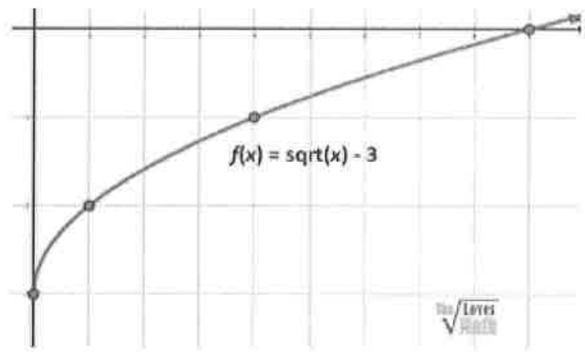
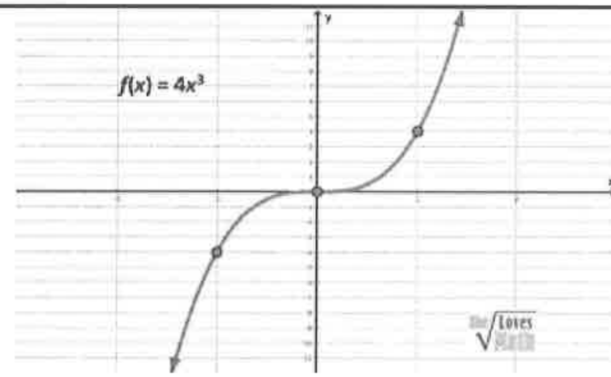
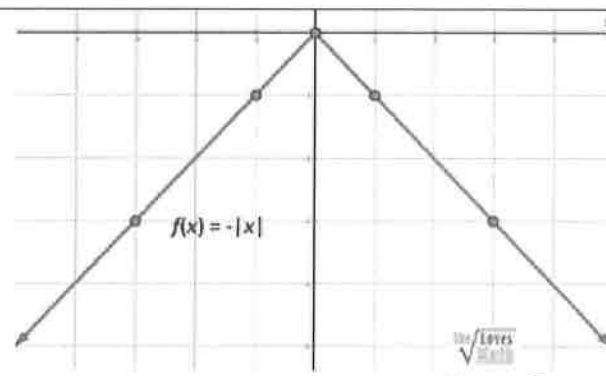


11. $|h(x)|$



12. $h(|x|)$

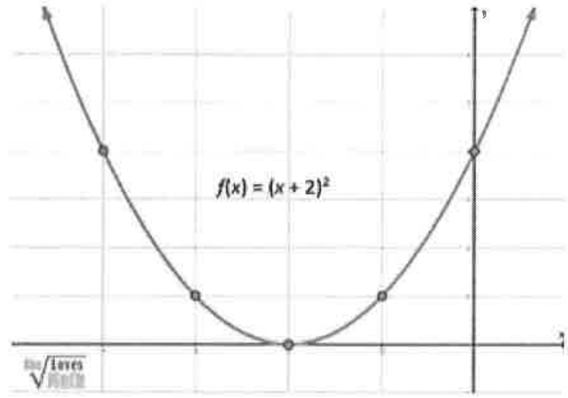


Trans-formation	What It Does	Example	Graph
$f(x)+b$	Move graph **up** b units Every point on the graph is shifted up **b** units. The ***x***'s stay the same; add **b** to the ***y*** values.	Parent: $y=x^2$ Transformed: $\mathbf{y=x^2+2}$ **x** | ***y*** | **y+2** −2 | 4 | 6 −1 | 1 | 3 0 | 0 | 2 1 | 1 | 3 2 | 4 | 6	 **Domain:** $(-\infty,\infty)$ **Range:** $[2,\infty)$
$f(x)-b$	Move graph **down** b units Every point on the graph is shifted down **b** units. The ***x***'s stay the same; subtract **b** from the ***y*** values.	Parent: $y=\sqrt{x}$ Transformed: $\mathbf{y=\sqrt{x}-3}$ **x** | ***y*** | ***y*−3** 0 | 0 | −3 1 | 1 | −2 4 | 2 | −1 9 | 3 | 0	 **Domain:** $[0,\infty)$ **Range:** $[-3,\infty)$
$a\cdot f(x)$	**Stretch** graph vertically by **a** (sometimes called a **dilation**) Every point on the graph is stretched **a** units. The ***x***'s stay the same; multiply the ***y*** values by **a**.	Parent: $y=x^3$ Transformed: $\mathbf{y=4x^3}$ **x** | ***y*** | **4y** −1 | −1 | −4 0 | 0 | 0 1 | 1 | 4	 **Domain:** $(-\infty,\infty)$ **Range:** $(-\infty,\infty)$
$-f(x)$	**Flip** graph around the ***x* axis** Every point on the graph is flipped vertically. The ***x***'s stay the same; multiply the ***y*** values by **−1**.	Parent: $y=\|x\|$ Transformed: $\mathbf{y=-\|x\|}$ **x** | ***y*** | **−y** −3 | 3 | −3 −1 | 1 | −1 0 | 0 | 0 1 | 1 | −1 3 | 3 | −3	 **Domain:** $(-\infty,\infty)$ **Range:** $(-\infty,0]$

<table>
<tr><th>Trans-
formation</th><th>What It Does</th><th>Example</th><th>Graph</th></tr>
<tr><td>$f(x+b)$</td><td>Move graph left b units

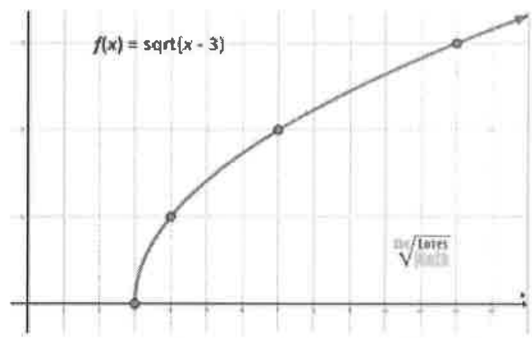
(Do the "opposite" when change is inside the parentheses or underneath radical sign.)

Every point on the graph is shifted left b units.

The y's stay the same; subtract b from the x values.</td><td>Parent: $y=x^2$
Transformed:
$y=(x+2)^2$
<table><tr><th>x −2</th><th>x</th><th>y</th></tr><tr><td>−4</td><td>−2</td><td>4</td></tr><tr><td>−3</td><td>−1</td><td>1</td></tr><tr><td>−2</td><td>0</td><td>0</td></tr><tr><td>−1</td><td>1</td><td>1</td></tr><tr><td>0</td><td>2</td><td>4</td></tr></table></td><td>

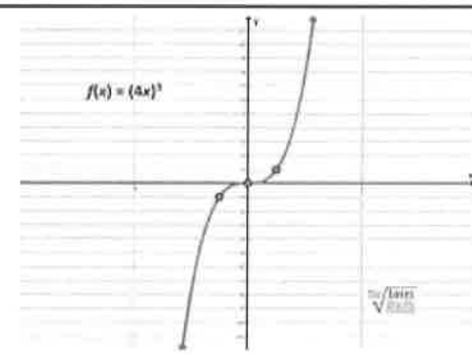
Domain: $(-\infty,\infty)$ Range: $[0,\infty)$</td></tr>
<tr><td>$f(x-b)$</td><td>Move graph right b units

Every point on the graph is shifted right b units.

The y's stay the same; add b to the x values.</td><td>Parent: $y=\sqrt{x}$
Transformed:
$y=\sqrt{x-3}$
<table><tr><th>x + 3</th><th>x</th><th>y</th></tr><tr><td>3</td><td>0</td><td>0</td></tr><tr><td>4</td><td>1</td><td>1</td></tr><tr><td>7</td><td>4</td><td>2</td></tr><tr><td>12</td><td>9</td><td>3</td></tr></table></td><td>

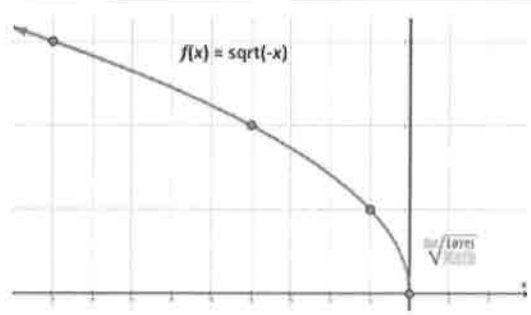
Domain: $[-3,\infty)$ Range: $[0,\infty)$</td></tr>
<tr><td>$f(a\cdot x)$</td><td>Compress (or squish) graph horizontally by a units (stretch or multiply by $\frac{1}{a}$)

Every point on the graph is compressed a units horizontally.

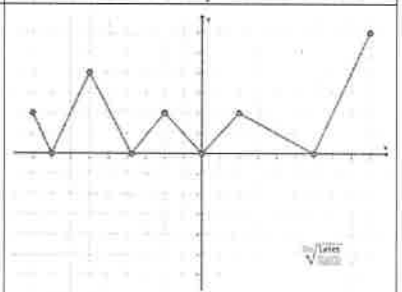
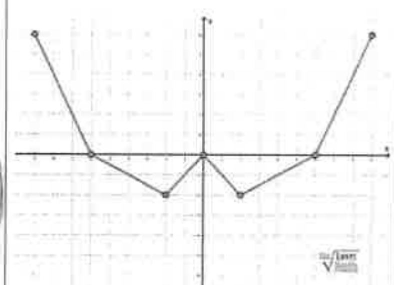
The y's stay the same; multiply the x values by $\frac{1}{a}$.</td><td>Parent: $y=x^3$
Transformed:
$y=(4x)^3$
<table><tr><th>$\frac{1}{4}x$</th><th>x</th><th>y</th></tr><tr><td>$-\frac{1}{4}$</td><td>−1</td><td>−1</td></tr><tr><td>0</td><td>0</td><td>0</td></tr><tr><td>$\frac{1}{4}$</td><td>1</td><td>1</td></tr></table></td><td>

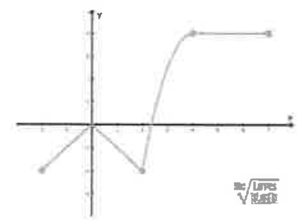
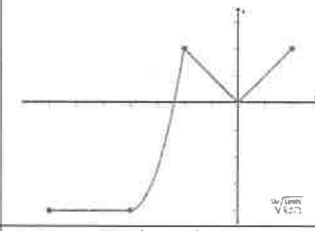
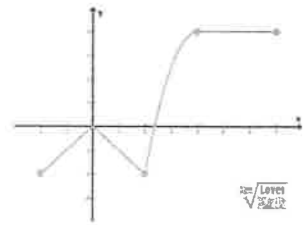
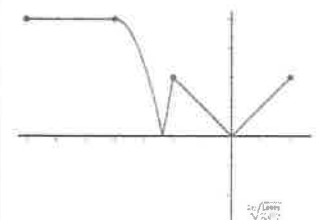
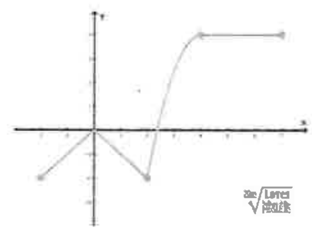
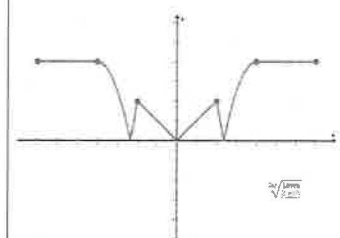
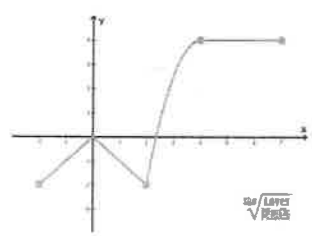
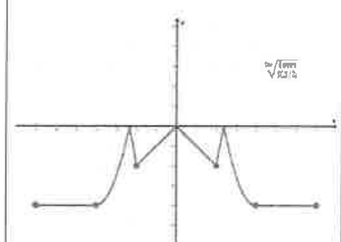
Domain: $(-\infty,\infty)$ Range: $(-\infty,\infty)$</td></tr>
<tr><td>$f(-x)$</td><td>Flip graph around the y axis

Every point on the graph is flipped around the y axis.

The y's stay the same; multiply the x values by −1.</td><td>Parent: $y=\sqrt{x}$
Transformed:
$y=\sqrt{-x}$
<table><tr><th>−x</th><th>x</th><th>y</th></tr><tr><td>0</td><td>0</td><td>0</td></tr><tr><td>−1</td><td>1</td><td>1</td></tr><tr><td>−4</td><td>4</td><td>2</td></tr><tr><td>−9</td><td>9</td><td>3</td></tr></table></td><td>

Domain: $(-\infty,0]$ Range: $[0,\infty)$</td></tr>
</table>

Transformation	T-chart	Graph
Original Function (Points)	<table><tr><th>x</th><th>y</th></tr><tr><td>−9</td><td>2</td></tr><tr><td>−6</td><td>−4</td></tr><tr><td>−2</td><td>2</td></tr><tr><td>0</td><td>0</td></tr><tr><td>2</td><td>−2</td></tr><tr><td>6</td><td>0</td></tr><tr><td>9</td><td>6</td></tr></table>	
$y = \|f(x)\|$ Replace all **negative *y* values** with their **absolute value** (make them positive). Make sure that **all** (negative ***y***) **points** on the graph are reflected across the ***x*** axis to be positive. (We had to use more points than the original to draw the correct graph.) Example Function: $y = \|x^3 + 4\|$	<table><tr><th>x</th><th>y</th></tr><tr><td>−9</td><td>2</td></tr><tr><td>−6</td><td>~~−4~~ 4</td></tr><tr><td>−2</td><td>2</td></tr><tr><td>0</td><td>0</td></tr><tr><td>2</td><td>~~−2~~ 2</td></tr><tr><td>6</td><td>0</td></tr><tr><td>9</td><td>6</td></tr></table>	
$y = f(\|x\|)$ Make a symmetrical graph from the positive ***x*****'s** across the ***y*** axis. "Throw away" the left hand side of the graph (negative ***x*****'s**), and **replace the left side** of the graph with the **reflection of the right hand side.** OR For any negative ***x*****'s**, replace the ***y*** value with the ***y*** value corresponding to the **positive value** (absolute value) of the negative ***x*****'s**. For example, when ***x*** is −9, replace the ***y*** with a 6, since the ***y*** value for **positive** 9 is 6. Example Function: $y = \|x - 4\|^3$	<table><tr><th>x</th><th>y</th></tr><tr><td>−9</td><td>~~2~~ 6</td></tr><tr><td>−6</td><td>~~−4~~ 0</td></tr><tr><td>−2</td><td>~~2~~ −2</td></tr><tr><td>0</td><td>0</td></tr><tr><td>2</td><td>−2</td></tr><tr><td>6</td><td>0</td></tr><tr><td>9</td><td>6</td></tr></table> 	

Original Function	Transformation	Explanation
	$-f(-x)$ 	Flip the function around the ***x*** axis, and then around the ***y*** axis. It actually doesn't matter which flip you perform first.
	$\|f(-x)\|$ 	Flip the function around the ***x*** axis, and then reflect everything below the ***x*** axis to make it above the ***x*** axis; this takes the absolute value. We actually could have done this in the other order, and it would have worked!
	$\|f(\|x\|)\|$ 	For the absolute value on the inside, throw away the negative ***x*** values, and replace them with the ***y*** values for the absolute value of the ***x***. Then reflect everything below the ***x*** axis to make it above the ***x*** axis; this takes the absolute value. We could have done this in any order.
	$-\|f(\|x\|)\|$ 	Do everything we did in the transformation above, and then flip the function around the ***x*** axis, because of the negative sign. For this one, I noticed that we needed to do the flip around the ***x*** axis last (we need to work "inside out").